universität freiburg

Stochastik I

Wintersemester 2025/2026

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Tutorial 1-Deadline: 03.02.2026

Exercise 1 (4 points)**.**
Let X be a random variable.

(a) Suppose that X is non-negative and has density function f. Show that for any $r \geq 1$ such that $\mathbb{E}(X^r) < \infty$,

$$\mathbb{E}(X^r) = r \int_0^\infty x^{r-1} \, \mathbb{P}(X > x) \, dx.$$

(b) Suppose that X is a continuous random variable and that $\mathbb{E}(|X|^r)$ exists for some integer $r \geq 1$. Show that

$$\int_0^\infty x^{r-1} \, \mathbb{P}(|X| > x) \, dx < \infty,$$

and that

$$x^r \, \mathbb{P}(|X| > x) \longrightarrow 0 \qquad \text{as } x \to \infty.$$

Hint: For part (a), use either integration by parts or Fubini's theorem (see https://en.wikipedia.org/wiki/Fubini's_theorem). For part (b), consider the positive part of X and distinguish between the cases where r is odd and where r is even.

Exercise 2 (4 points)**.**
Let $X \sim \mathrm{Geo}(p)$ be geometrically distributed with parameter $p \in (0,1)$, that is,

$$\mathbb{P}(X = k) = p(1-p)^{k-1}, \qquad k \in \mathbb{N}.$$

(a) Compute $\mathbb{E}[X]$.

(b) Compute $\mathrm{Var}(X)$.

(c) Show that X is *memoryless*, that is, for all $k, l \in \mathbb{N}$,

$$\mathbb{P}(X > k + l \mid X > l) = \mathbb{P}(X > k).$$

Exercise 3 (4 points)**.**
Let $X \sim \mathrm{Exp}(\lambda)$ be exponentially distributed with parameter $\lambda > 0$, that is, X has density

$$f_X(x) = \lambda e^{-\lambda x}, \qquad x \geq 0.$$

(a) Compute $\mathbb{E}[X]$.

(b) Compute $\mathrm{Var}(X)$.

(c) Show that X is *memoryless*, that is, for all $s,t \geq 0$,

$$\mathbb{P}(X > s + t \mid X > t) = \mathbb{P}(X > s).$$

Exercise 4 (4 points)**.**
Let X and Y be independent exponential random variables with parameter 1. Define

$$U = X + Y, \qquad V = \frac{X}{X + Y}.$$

1. Find the joint density function of (U,V).
2. Deduce that V is uniformly distributed on $[0,1]$.
3. Identify the distribution of U.

Hint: Use the change-of-variables formula for probability densities. Compute the Jacobian of the inverse transformation to obtain the joint density of the new variables. (see https://en.wikipedia.org/wiki/Probability_density_function#Function_of_random_variables_and_change_of_variables_in_the_probability_density_function).